Journal of Engineering Physics and Thermophysics, Vol. 81, No. 6, 2008

FEATURES OF THE DISTRIBUTION OF HEAT FLUXES IN THE ANODE–VAPOR-GAS SHEATH SYSTEM IN ANODIC ELECTROLYTIC HEATING

P. N. Belkin, T. L. Mukhacheva,
and I. G. D'yakov

UDC 621.785.53;621.3.035.183

An expression for the density of the heat flux acting between a vapor-gas sheath and a vertical cylindrical anode has been obtained from solution of the heat-conduction equation in the vapor-gas sheath and experimental measurements of he anode-specimen temperature. It has been shown that the heat flux from the sheath to the anode acts just in the lower part of the system. In its upper part, heat fluxes are directed from the anode to the sheath, which limits the possibility of applying the heat-transfer coefficient to description of heat exchange in anodic heating. The values of the vertical temperature gradient in heating of the cylindrical anode and the fraction of heat coming into it have been measured.

Introduction. Heating of the anode in aqueous electrolytes results from the local boiling of the electrolyte, which gives rise to a solid and stable vapor-gas sheath (VGS). This sheath possesses intrinsic electrical conductivity and is a primary heat source in the system. It has been assumed that stationary heat exchange between the heated anode and the VGS can be described by the Newton–Richmann law with determination of the heat-transfer coefficient by the method of regular regime of the first kind [1]. Vertically oriented cylindrical specimens made of 45 steel with a diameter of 10 to 30 mm were heated under the conditions of longitudinal electrolytic flow with velocities of 0.2 to 0.6 cm/sec. The specimens were submerged in the aqueous solution containing 11% ammonium chloride and 11% ammonium nitrate to a depth of 26 mm. The electrolyte temperature was held equal to 30 ± 2^{o}C. The specimens' temperature was measured with a Chromel-Alumel thermocouple arranged in the blind axial hole of the specimen; the thermocouple signal was fed to the recorder. Heating and cooling were carried out by increasing or decreasing abruptly the voltage, which caused the low-inertia VGS to quite rapidly change. The oscillograms of the current showed that the thickness of the sheath and, apparently, the temperature distribution in it are stabilized in a matter of milliseconds. The presence of the regular regime on the curves of heating and cooling of the specimens enabled us to evaluate the coefficient of heat transfer from the sheath to the anode at 1400 ± 200 W/(m^2·K).

Also, the coefficient of heat transfer between the VGS and the anode was determined for nonstationary conditions in heating of the end of a cylindrical specimen of diameter 20 mm by a submerged electrolyte jet [2]. The anode-specimen temperature and the current were recorded by a potentiometer; the heat flux to the anode was found by solution of the inverse heat-conduction problem for the heated rod by the least-squares method [3]. The obtained value of the heat-transfer coefficient was 2500 ± 500 W/(m^2·K) with the refinement that the surface of maximum temperatures in the VGS refers to the arbitrary heat-release surface.

The significant difference in the published data is attributable to the different geometries of the systems studied. The height-average heat-transfer coefficient determined by the regular-regime method describes heat exchange on the vertical cylindrical surface of the anode specimen. Here, it has been assumed that heat exchange on the specimen's end can be disregarded. The second result has been obtained in heating of a flat surface by an electrolyte jet, i.e., under different hydrodynamic conditions.

In both cases we disregarded the nonuniform energy release in the VGS on the heat-exchange surface, i.e., the nonuniform temperature distribution. Meanwhile, it has been established that the thickness of the anode sheath in heating of vertical rods expands in the upward direction [4]. The current-density distribution on the surface of contact of the

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N. A. Nekrasov Kostroma State University, 14 1 Maya Str., Kostroma, 156961, Russia; belkinp@yandex. ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 81, No. 6, pp. 1027–1033, November–December, 2008. Original article submitted March 13, 2007; revision submitted September 21, 2007.

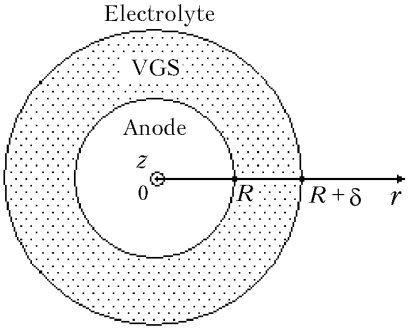


Fig. 1. Diagram of the domain of solution of the heat-conduction equation in the anode VGS.

electrolyte jet with the heated flat surface is nonuniform, too, which causes the heating temperature to decrease from the center of the heated portion to its periphery [5]. In our opinion, the reason is the change in the VGS temperature from the minimum temperature at the center to the maximum one at the edges of the heated portion. The presence of the temperature gradient in heating of vertically oriented parts has been noted for both the cathodic variant of heating [6] and the anodic variant [7], which imposes restrictions on their application to thermal and chemical-thermal treatment. According to our measurements, the dependence of the temperature of vertically submerged specimens on the vertical coordinate is linear, and the average value of the temperature gradient can range from 2 to 20°C/mm.

This work seeks to analyze the heat fluxes between the VGS and the anode with allowance for the vertical temperature gradient in the heated cylindrical specimens.

Theoretical Analysis. Let us consider the temperature distribution in the VGS surrounding a vertical cylindrical anode (Fig. 1). In a thin vapor layer, along the horizontal radial coordinate, it is found from the heat-conduction equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{W}{\lambda}, \quad R \le r \le R+\delta . \tag{1}$$

The specific heat-source strength in the sheath with allowance for space changes is determined, according to [8], by the expression

$$W=\frac{9\mu\varepsilon\varepsilon_0 U^3}{8\delta^4} . \tag{2}$$

Equation (1) is supplemented by the boundary conditions:

(1) the temperature at the vapor-liquid interface is equal to the saturation temperature of the electrolyte $T(R+\delta) = T_s$;

(2) the heat flux from the sheath to the anode is determined by the Newton–Richmann law: $\lambda\frac{\partial T}{\partial r}\Big|_{r=R} = \alpha[T_m - T(R)]$.

We note that the saturation temperature of the aqueous solution of ammonium chloride of the concentration used is no more than 2°C higher than the saturation temperature of water.

The solution of (1) has the form

$$T(r)=-\frac{W}{4\lambda}r^2 + C_1 \ln r + C_2 . \tag{3}$$

Finding the constant C_1 and C_2 directly from the boundary conditions leads to a transcendental equation for determination of the maximum VGS temperature T_m. The problem is simplified by linearization of Eq. (3). For this purpose we use logarithmic expansion with allowance for the fact that the sheath thickness is two to three orders of magnitude smaller than the anode-specimen radius:

$$\ln r = \ln \frac{r}{R} + \ln R = \ln \frac{R+\delta}{R} + \ln R = \frac{r}{R} - 1 + \ln R\,.$$

Then the maximum sheath temperature determined from the equality of the derivative of the linearized temperature distribution to zero is as follows:

$$T_{\mathrm{m}} = C_1\,[\ln R - 1] + C_2 + C_1^2 \frac{\lambda}{WR^2}\,. \tag{4}$$

Substitution of (4) into the second boundary condition yields the quadratic equation for the constant C_1, whence we have

$$C_1 = \frac{WR}{2}\left(\frac{R}{\lambda} + \frac{2}{\alpha}\right).$$

The second root of the solution corresponds to the zero heat flux to the anode; therefore, it is not considered. The constant C_2 is found from the first boundary condition to Eq. (1). Thus, the radial temperature distribution in the VGS has the form

$$T(r) = T_{\mathrm{s}} + \frac{W(R+\delta)^2}{4\lambda} - \frac{W}{4\lambda}r^2 + \frac{WR}{2}\left(\frac{R}{\lambda} + \frac{2}{\alpha}\right)\ln\left(\frac{r}{R+\delta}\right). \tag{5}$$

We note that the solution (5) satisfies the second boundary condition accurate to $\lambda/(\alpha R) \approx 0.01$.

To allow for the vertical temperature gradient in the sheath we consider heat balance in a thin anode specimen submerged in the electrolyte to a depth equal to its length h. Three heat fluxes act in the volume element of the anode from z to $z + dz$. The fluxes through the cross sections at the heights z and $z + dz$ due to the heat conduction along the anode will be equal to

$$dQ_z = -\lambda_{\mathrm{a}} \frac{dT_{\mathrm{a}}}{dz}(z)\,\pi R^2\,dt\,,\quad dQ_{z+dz} = -\lambda_{\mathrm{a}} \frac{dT_{\mathrm{a}}}{dz}(z+dz)\,\pi R^2 dt\,. \tag{6}$$

To them is added the heat flux acting from the VGS through the lateral surface by the Newton–Richmann law:

$$dQ_{\mathrm{v}} = \alpha 2\pi R dz\,(T_{\mathrm{m}} - T_{\mathrm{a}})\,dt\,. \tag{7}$$

These fluxes are related by the heat-balance equation

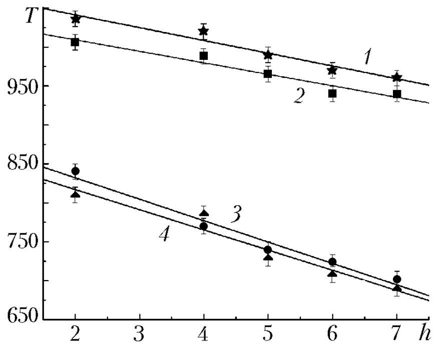
$$dQ_z + dQ_{\mathrm{v}} - dQ_{z+dz} = 0\,. \tag{8}$$

Substitution of (6) and (7) into (8) leads, on rearrangement, to the equation of temperature distribution in the anode along the vertical coordinate:

$$\frac{d^2T_{\mathrm{a}}}{dz^2} + \frac{2\alpha}{\lambda_{\mathrm{a}}R}(T_{\mathrm{m}} - T_{\mathrm{a}}) = 0\,. \tag{9}$$

The density of the heat flux from the sheath to the anode is equal, with account for (5), to

$$q = -\lambda \left.\frac{dT}{dr}\right|_{r=R} = \frac{\lambda W}{\alpha} = \alpha\,(T_{\mathrm{m}} - T_{\mathrm{a}})\,. \tag{10}$$



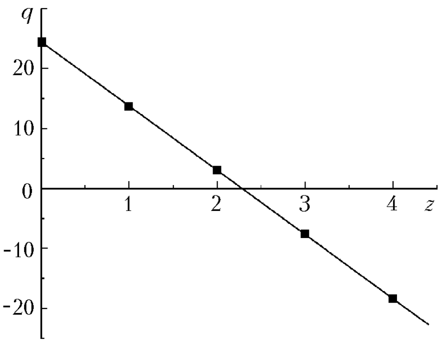


Fig. 2. Influence of the length of vertical cylindrical specimens on their axial temperature measured at the lower (1 and 2) and upper (3 and 4) points of the specimens. The heating voltage is 260 V (1 and 3) and 200 V (2 and 4). T, ^{o}C; h, cm.

Fig. 3. Density of the heat flux through the lateral surface of the anode of length 4 cm, $U = 280$ V. q, MW/m^2; z, cm.

Then, according to (10), Eq. (9) will take the following form:

$$\frac{d^2 T_a}{dz^2} + \frac{2q}{\lambda_a R} = 0 \, . \tag{11}$$

The temperature distribution on the anode surface, which is obtained by substitution of $r = R$ into (5), offers the second equation relating the anode temperature to the density of the heat flux from the sheath to the anode. Replacing the heat-transfer coefficient by its expression by the heat-flux density (10), we obtain

$$T_a = T(R) = \frac{W\delta}{4\lambda}(2R + \delta) + \frac{WR}{2\lambda}\left(R + \frac{2q}{W}\right)\ln\left(\frac{R}{R + \delta}\right) + T_s \, . \tag{12}$$

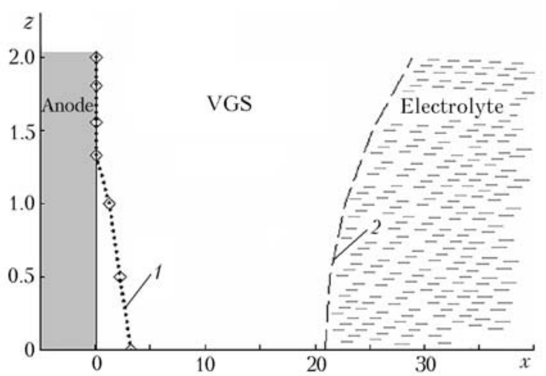
Experiment. System (11) and (12) enables us to determine the q and δ values as functions of the vertical coordinate from the experimental data of $T_a(z)$. We measured the temperature distribution with a step of 5 mm along the axis of vertically submerged 45-steel specimens of diameter 10 mm whose length ranged from 20 to 70 mm. The electrolyte flowed past the specimens with a rate of 2.5 liter/min; the composition of the electrolyte was a 10% aqueous solution of ammonium chloride. The electrolyte temperature measured at entry into the working chamber was stabilized at a level of 20^{o}C in heating of 20-mm-long specimens. The stable electrolyte temperature increased with length and was 27^{o}C for 70-mm-long specimens. The heating voltage ranged from 180 to 300 V.

Figure 2 gives results of measurements of the temperature of different points of the specimens. The experimental dependences of the anode temperature on the coordinate z were approximated by polynomials of the third degree with a correlation coefficient no less than 0.98:

$$T_a(z) = \sum_{i=0}^{3} b_i z^i \, , \tag{13}$$

where the empirical constants b_i were selected by the least-squares method. Substituting then the second derivative of (13) into (11), we obtained the dependence of the heat flux through the lateral anode surface on the vertical coordinate.

It turned out that the density of the heat flux from the sheath to the anode decreases in the vertical direction; for a certain value of the coordinate, it changes sign (Fig. 3). This result is consistent with the data obtained earlier in calculating of the temperature field of a vertically heated cylindrical anode [7]. Figure 4 gives a diagram of the anode VGS, where the auxiliary radial coordinate x is reckoned from the anode surface. The surface of maximum temperatures is offset by no more than 5 μm from the anode, which is no higher then 20% of the limiting sheath thick-



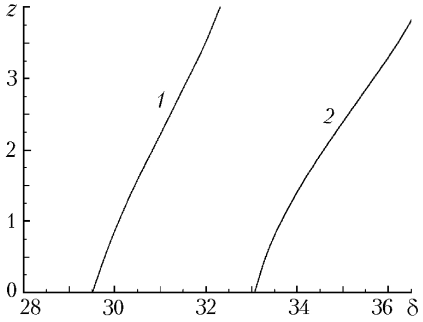


Fig. 4. Maximum-temperature surface (1) and the VGS profile (2) for the anode of length 2 cm at the voltage $U = 240$ V. z, cm; x, μm.

Fig. 5. Profile of the VGS for the anode of length 4 cm at $U = 200$ V (1) and 260 V (2). z, cm; δ, μm.

ness. For this reason the coincidence of the maximum-temperature surface with the actual surface of the anode, which is used in certain models, may be considered as a justified simplification. No dependence of the heat-flux density on the heating voltage has been revealed in the studied range.

Change in the direction of the heat flux to the VGS limits the applicability of the second boundary condition to (1) and of relation (10) to the coordinate region $z < z_0$. Therefore, to calculate the anode-specimen temperature in the upper part of the sheath we solved Eq. (1) with another boundary condition:

$$q = \lambda \left. \frac{dT}{dr} \right|_{r=R} = \alpha_{\text{up}} (T_{\text{a}} - T_{\text{s}}) , \tag{14}$$

where the heat-transfer temperature α_{up} describes the removal of heat from the anode specimen to the electrolyte surface with saturation temperature T_{s}.

Calculation of the anode-specimen temperature throughout its length enables us to determine the profile of the anode sheath for any value of the vertical coordinate from 0 to h (Fig. 5). Increase in the average sheath thickness with voltage has been confirmed. The sheath thickness at the anode base is 25–30 μm, i.e., is consistent with the results of [9–11] as far as the order of magnitude is concerned [9–11]. Increase in the sheath thickness with anode length is most likely due to the rise in the average electrolyte temperature.

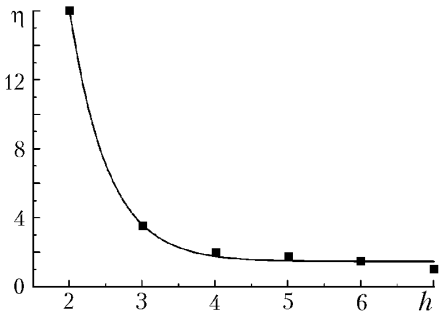
The dependence of the heat flux from the sheath to the anode on the voltage and the anode length is determined according to the expression

$$Q (h, U) = \int_0^h q (z) \, dz .$$

The following fraction of heat released in the system comes into the anode:

$$\eta = \frac{\int_0^h q (z) \, dz}{IU} . \tag{15}$$

Numerical evaluations show that, as the length of the specimens increases, the fraction of heat coming from the VGS into them diminishes (Fig. 6). This result is due to the fact that the heat coming from the sheath to the lower part of the anode partially returns back through the upper part of the anode. Such exchange diminishes the frac-



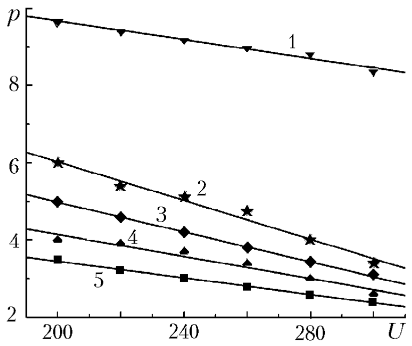


Fig. 6. Fraction of heat coming into the anode vs. its length. η, %; *h*, cm.

Fig. 7. Vertical temperature gradient vs. heating voltage. (Figures at the curves, specimens' length in centimeters.) *p*, K/mm; *U*, V.

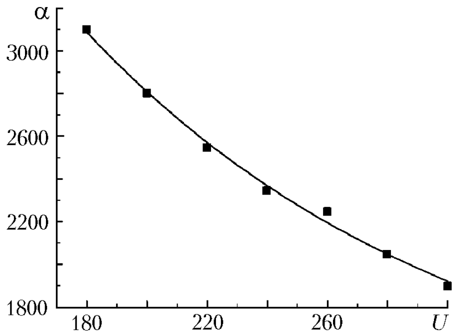


Fig. 8. Average heat-transfer coefficient vs. voltage for the anode of length 2 cm. α, W/(m^2·K); *U*, V.

tion of heat coming from the sheath into the anode, the more strongly, the greater the anode length. As it increases, the temperature gradient of the anode decreases vertically (Fig. 7).

Calculation of the coefficient of heat transfer from the upper part of the anode to the VGS from formula (14) yields values of 300–600 W/(m^2·K). Calculation of the coefficient of heat transfer from the VGS to the lower part of the anode from formula (10) leads to values of the order of 10^5 W/(m^2·K). In our opinion, the result obtained points to the inapplicability of the heat-transfer coefficient to the description of heat exchange in the anode–sheath–electrolyte system. In such a case the above-noted significant differences in experimental data are attributable primarily to the arbitrariness of the measured quantity. In anodic heating, we do not have a pronounced heat-release surface; therefore, it is allowable to use just the arbitrary heat-transfer coefficient. Formal averaging of the obtained heat-transfer coefficients over the entire anode length yields values comparable to those measured earlier (Fig. 8).

This work was carried out with financial support from the Ministry of Education and Science of the Russian Federation within the framework of the thematic plan of scientific-research works.

CONCLUSIONS

1. We have calculated the profile of the anode VGS on the vertical cylindrical specimen. The expansion of the sheath with increase in the heating voltage and in the specimen length has been found.

2. We have shown that the surface with maximum temperatures in the VGS is at a distance from the anode only in its lower part and is coincident with the surface of the anode in its upper part. Different directions of the heat fluxes between the sheath and the anode in the lower and upper parts of the system make it impossible to use the heat-transfer coefficient in the generally accepted sense for description of heat exchange in anodic heating.

3. We have found a decrease of 16 to 1% in the fraction of heat coming into the anode, as its length increases from 2 to 7 cm.

4. We have measured the values of the vertical temperature gradient of the anode; they are 2 to 20°C/mm. It has been established that the temperature gradient increases with increase in the voltage and decrease in the length of cylindrical specimens.

NOTATION

b_i, empirical constants, K/m^i; C_1 and C_2, constants; h, submerged length of the anode, m; I, current strength, A; p, vertical temperature gradient over the specimen's length, K/m; q, density of the heat flux acting between the VGS and the anode, W/m^2; Q_z and Q_{z+dz}, heat fluxes acting in the anode specimen, W; Q, heat flux from the sheath to the anode, W; Q_{v}, heat flux from the VGS to the anode specimen, W; r, radial coordinate reckoned from the anode-specimen axis, m; R, anode radius, m; t, time, sec; T, temperature in the VGS, K; T_{m}, maximum temperature in the sheath, K; T_{a}, anode temperature, K; T_{s}, saturation temperature of the solution, K; U, heating voltage, V; W, specific strength of the heat sources, W/m^3; x, radial coordinate reckoned from the anode surface, m; z, vertical coordinate, m; z_0, vertical coordinate for which the density of the heat flux from the sheath to the anode changes sign, m; α, average coefficient of heat transfer from the sheath to the anode, W/(m^2·K); α_{up}, coefficient of heat transfer from the upper part of the anode to the VGS, W/(m^2·K); δ, VGS thickness, m; ε, dielectric constant; ε_0, electric constant, F/m; η, fraction of heat coming into the anode from the VGS, %; λ, thermal conductivity of the vapor, W/(m·K); λ_{a}, thermal conductivity of the anode material, W/(m·K); μ, ion mobility, m^2/(W·sec). Subscripts: a, anode; m, maximum; s, saturation; v, vapor; up, upper part, top; i, summation.

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